Alternative approach to fit irregular corneas

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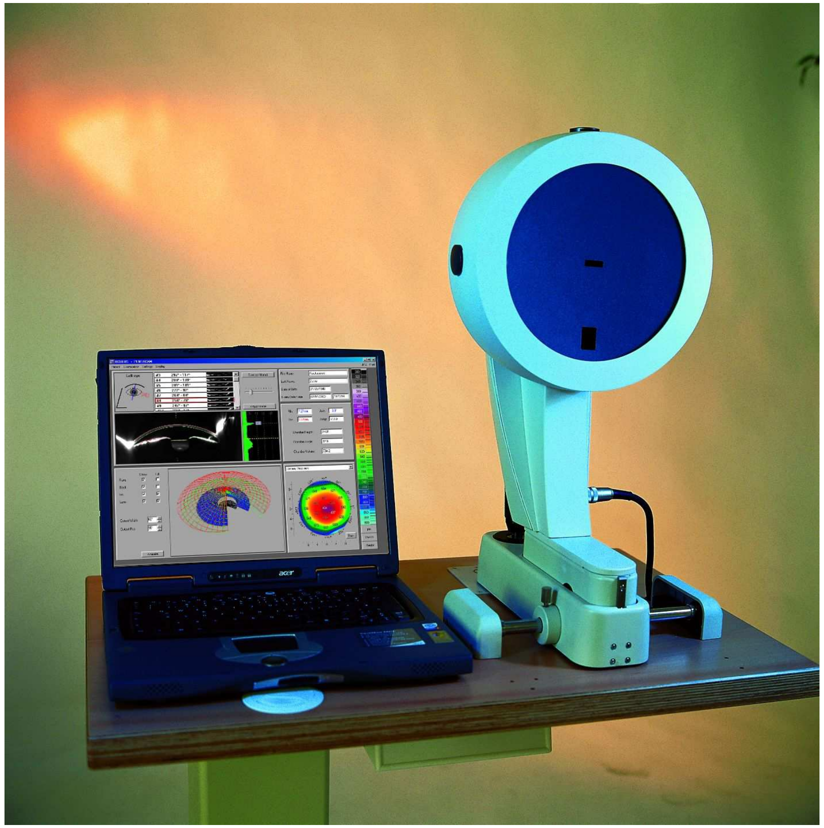
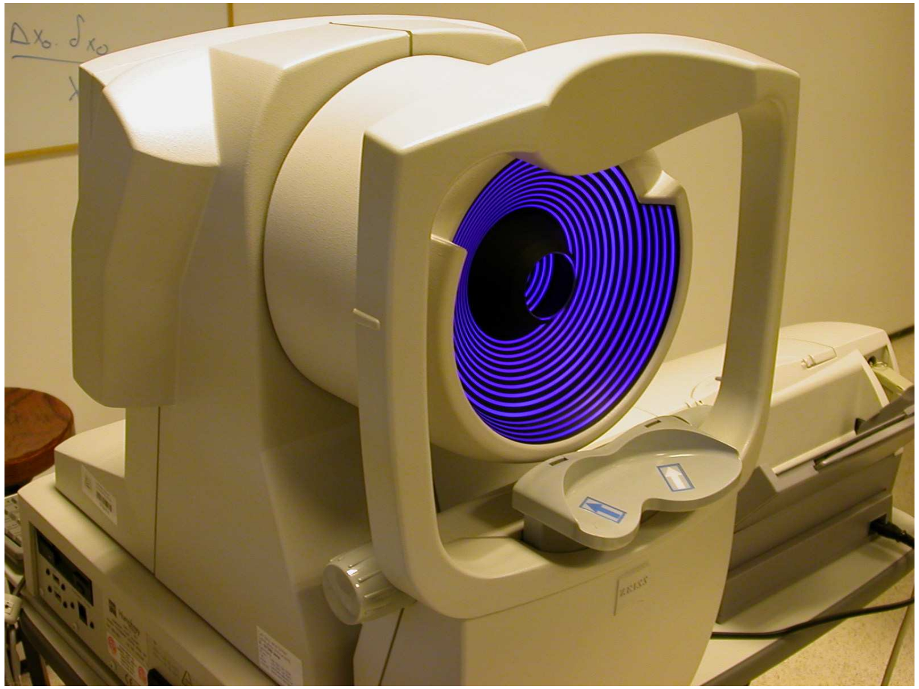
Background

Scheimpflug cameras and topographers based in Placido rings permit accurate estimation of corneal surfaces





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Background

Corneal height data ➔ Model ➔ Wavefront analysis

- Zonal approach.

    B-Splines

    Well suited for fitting complex-shaped surfaces
    Not related with aberrations

- Modal approach.

    Zernike polynomials

    Direct relation with Seidel aberrations
    Not precise when describing highly irregular corneas

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Objective

COMBINATION

ZONAL + MODAL approaches

Zonal Zernike fitting of corneal height data

Zernike coefficients computed in overlapping local areas

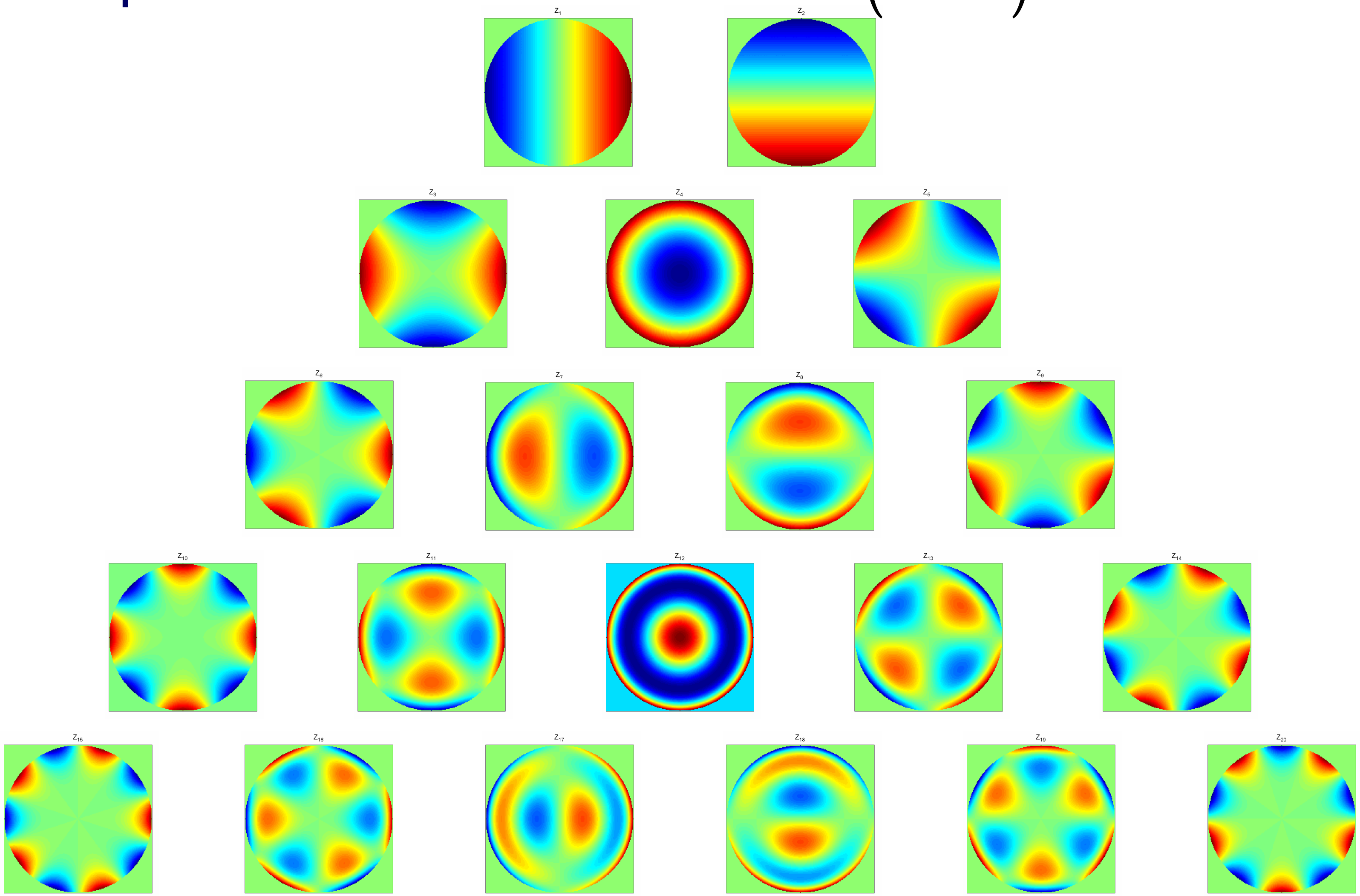
- Diminishing the influence of smooth areas over irregular zones and vice versa.
- Limiting the influence of the peripheral irregularities over the central corneal area, thus giving accurate reconstruction of the central optical zone.

Modal approach. Zernike polynomials

$$W\left(x_u, y_v\right) \simeq \sum_{j=0}^{p-1} c_j Z_j\left(x_u, y_v\right)$$

Least-squares method

$$C = \left(\mathbf{Z}^T \mathbf{Z}\right)^{-1} \mathbf{Z}^T \mathbf{W}$$



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Modal approach. Zernike polynomials

$$W\left(x_u, y_v\right) \simeq \sum_{j=0}^{p-1} c_j Z_j\left(x_u, y_v\right)$$

Least-squares method

$$C = \left(\mathbf{Z}^T\mathbf{Z}\right)^{-1}\mathbf{Z}^T\mathbf{W}$$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ \ldots \\ c_{p-2} \\ c_{p-1} \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} \sum_{j=0}^{p-1} Z_j\left(x_1, y_1\right) & \ldots & \sum_{j=0}^{p-1} Z_j\left(x_n, y_1\right) \\ \ldots & \ldots & \ldots \\ \sum_{j=0}^{p-1} Z_j\left(x_1, y_n\right) & \ldots & \sum_{j=0}^{p-1} Z_j\left(x_n, y_n\right) \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W\left(x_1, y_1\right) & W\left(x_2, y_1\right) & \ldots & W\left(x_{n-1}, y_1\right) & W\left(x_n, y_1\right) \\ W\left(x_1, y_2\right) & W\left(x_2, y_2\right) & \ldots & W\left(x_{n-1}, y_2\right) & W\left(x_n, y_2\right) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ W\left(x_1, y_{n-1}\right) & W\left(x_2, y_{n-1}\right) & \ldots & W\left(x_{n-1}, y_{n-1}\right) & W\left(x_n, y_{n-1}\right) \\ W\left(x_1, y_n\right) & W\left(x_2, y_n\right) & \ldots & W\left(x_{n-1}, y_n\right) & W\left(x_n, y_n\right) \end{bmatrix}$$

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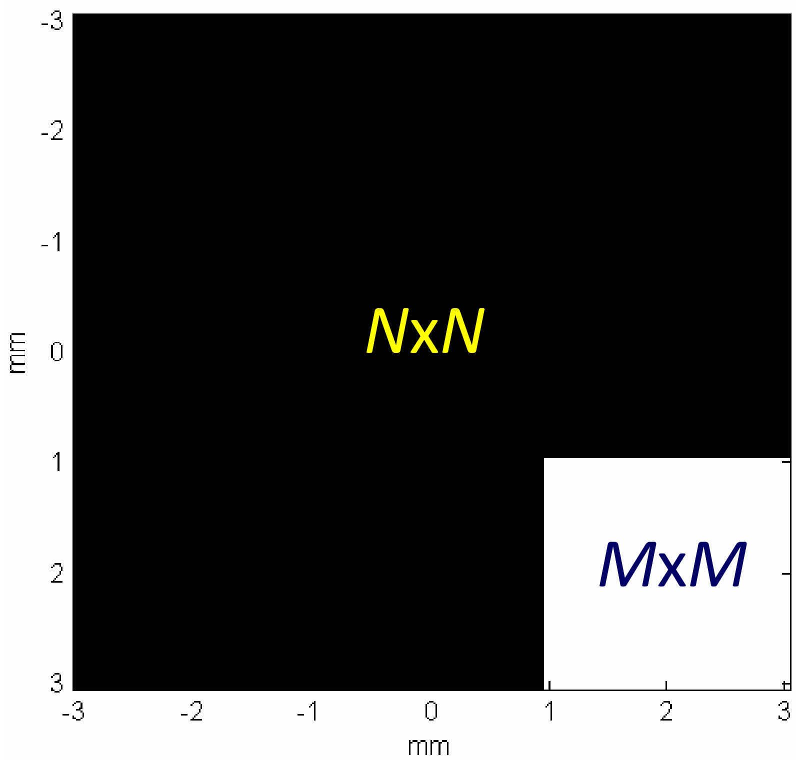
Zonal Zernike fitting

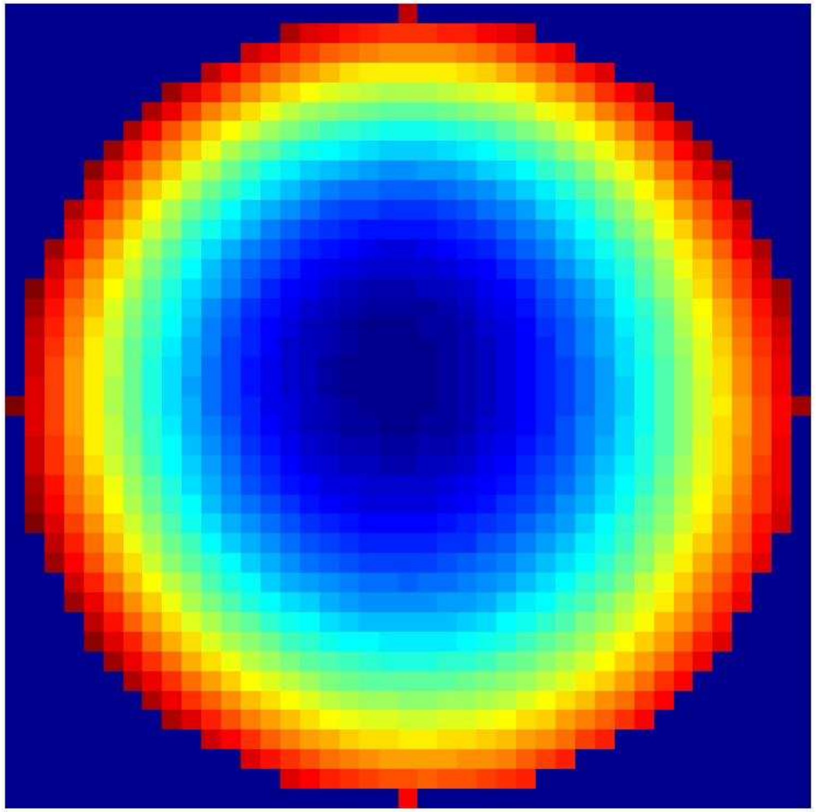
Auxiliary *a* x *b* matrices of size N×N, unmasking MxM *zone*

$$W_{a,b}(u,v) \simeq \begin{cases} \sum_{j=0}^{p-1} c_j^{(a,b)} Z_j\left(\rho_{(u,v)}, \theta_{(u,v)}\right); \\ 0; \end{cases}$$



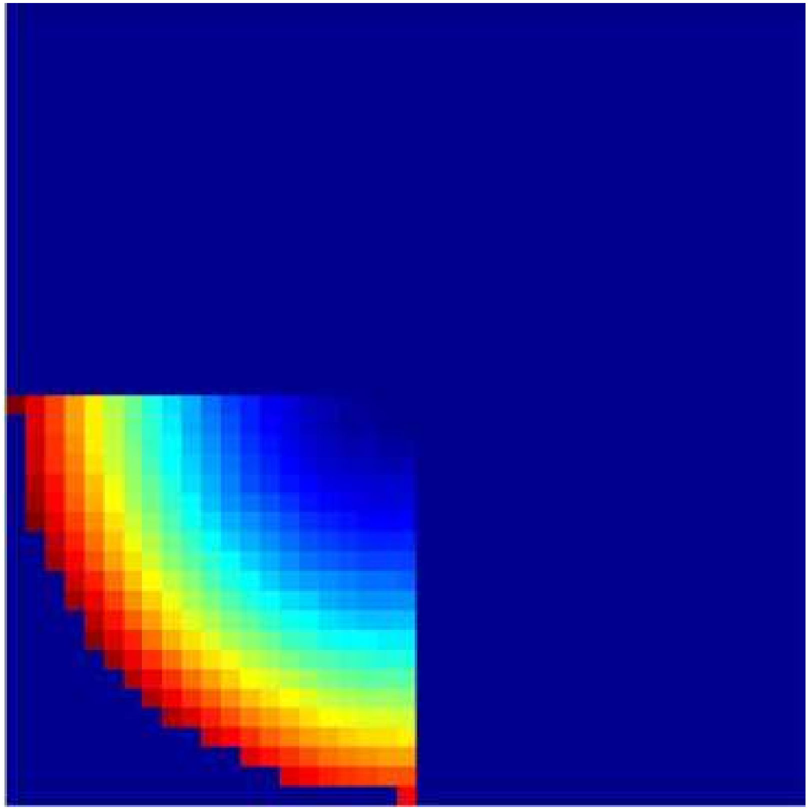
Zonal Zernike fitting

Modal



(NxN)

Modal+Zonal



(MxM)

One point of the surface may belong to different local regions

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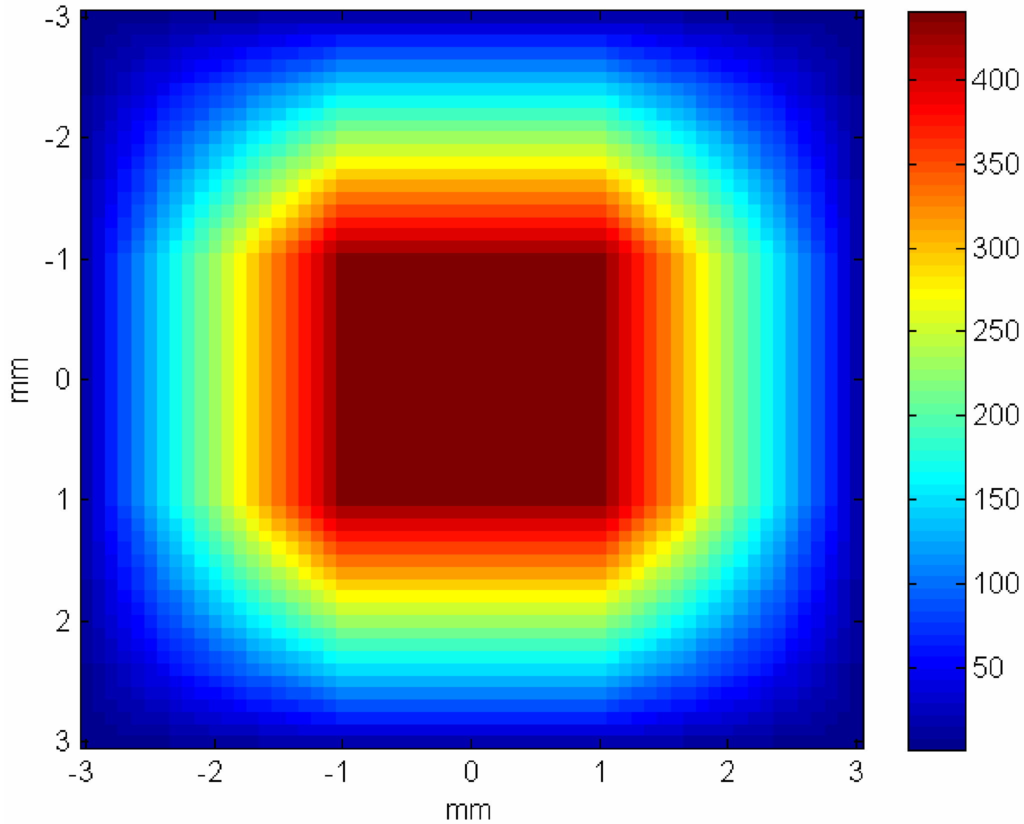
Zonal Zernike fitting

Reconstructed surface:
Mean at each point

$$L(u,v)=\frac{\sum_{a=1}^{N-(M-1)}\sum_{b=1}^{N-(M-1)}W_{a,b}(u,v)}{\sum_{a=1}^{N-(M-1)}\sum_{b=1}^{N-(M-1)}O_{a,b}(u,v)}$$

$$\sum_{a=1}^{N-(M-1)}\sum_{b=1}^{N-(M-1)}O_{a,b}(u,v)$$



Points in the central zone are evaluated M^2 times

M=21 px ➔ 441 times

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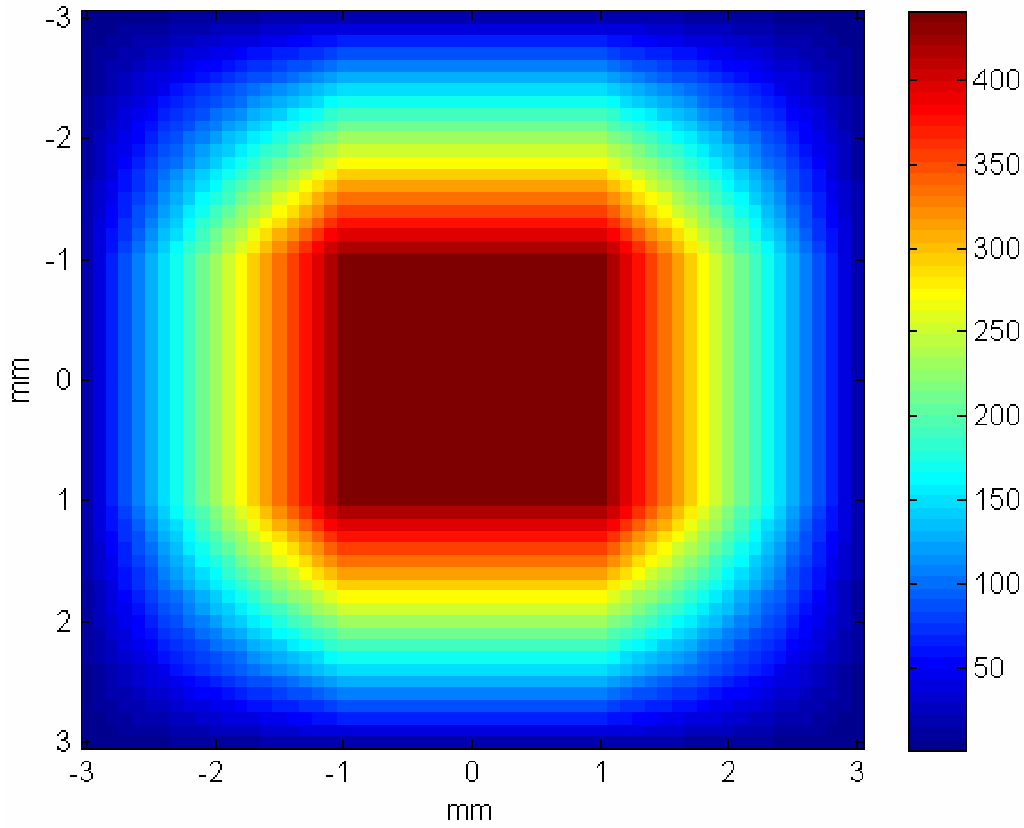
Zonal Zernike fitting

Number of elements in the central zone

$$f_{\mathrm{N}}(\mathbf{M}) = \mathbf{M}^2\left[\mathbf{N} - 2(\mathbf{M}-1)\right]^2$$



From the derivative:

$$\mathbf{M}_{\mathrm{opt}} = \mathrm{round}\left(\frac{\mathbf{N}+2}{4}\right)$$

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Results

- Height data analysis of surfaces.
  - Irregular surface
  - Keratoconus from Pentacam

Root mean square deviation (RMSD)

w_i = original surface data

g_i = reconstructed surface data

$$RMSD = \sqrt{\frac{\sum_{i=1}^{N'} \left(w_i - g_i\right)^2}{N'}}$$

- Validate M_{opt}. Two masks of different sizes

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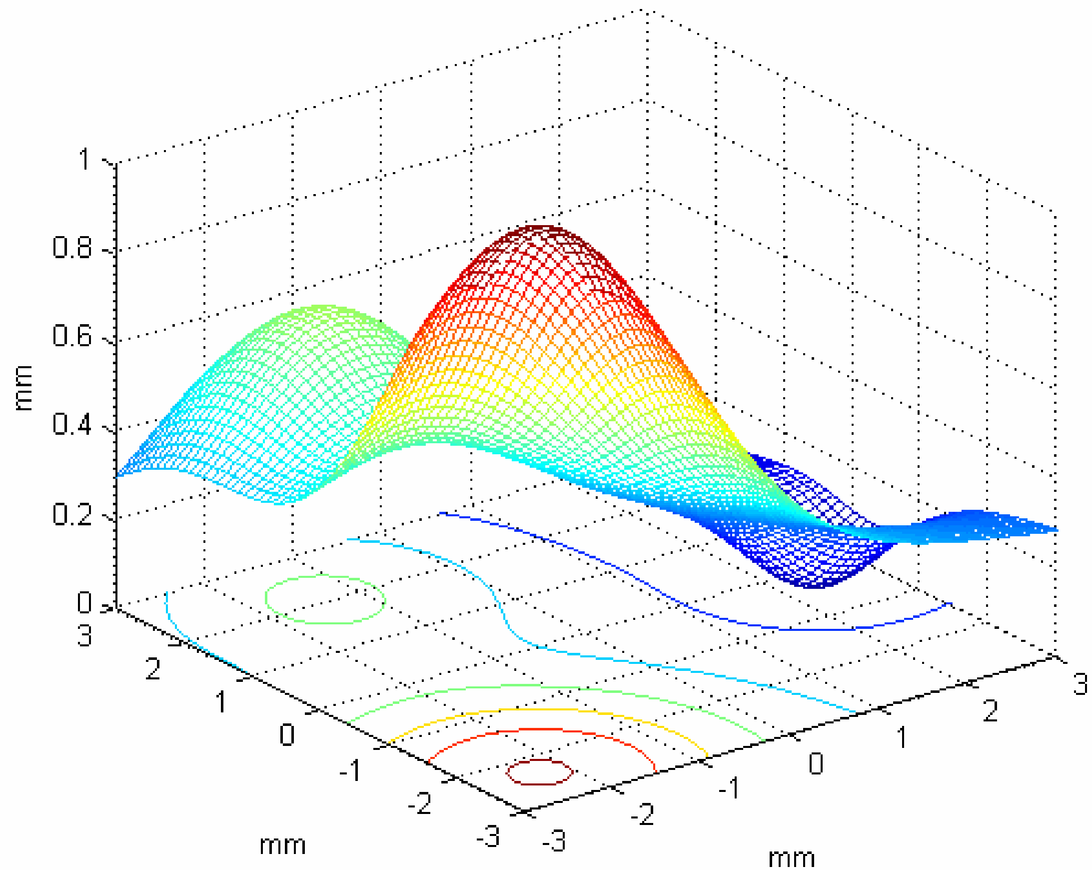
Irregular surface: Sphere+Franke's function

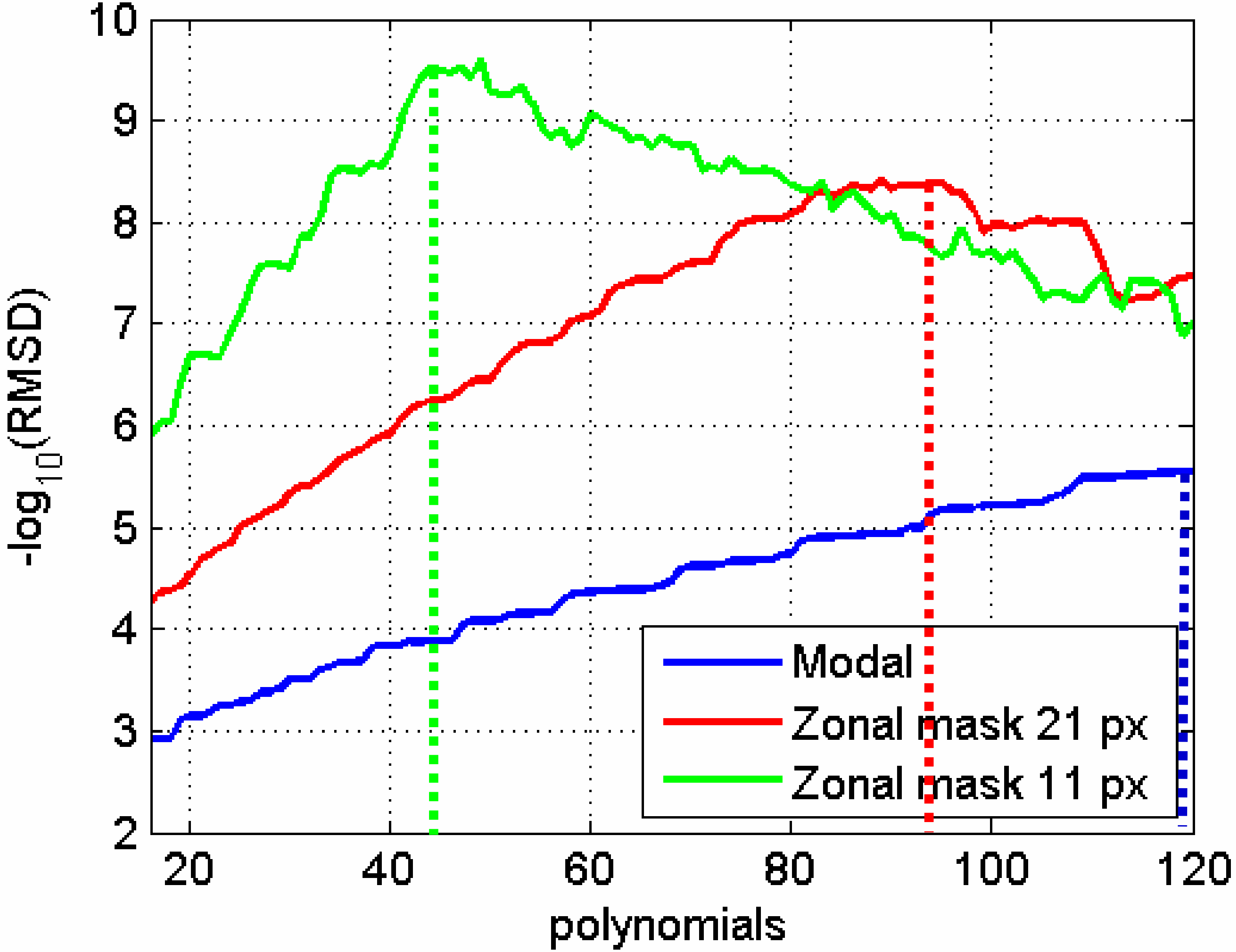
$$fr(x,y)=\frac{3}{4}\exp\left[\frac{-\left((9x-2)^2+(9y-2)^2\right)}{4}\right]+\frac{3}{4}\exp\left[-\left(\frac{(9x+1)^2}{49}+\frac{(9y+1)^2}{10}\right)\right]$$

$$+\frac{1}{2}\exp\left[-\left((9x-7)^2+\frac{(9y-3)^2}{4}\right)\right]+\frac{1}{5}\exp\left[-\left((9x-4)^2+(9y-7)^2\right)\right]$$



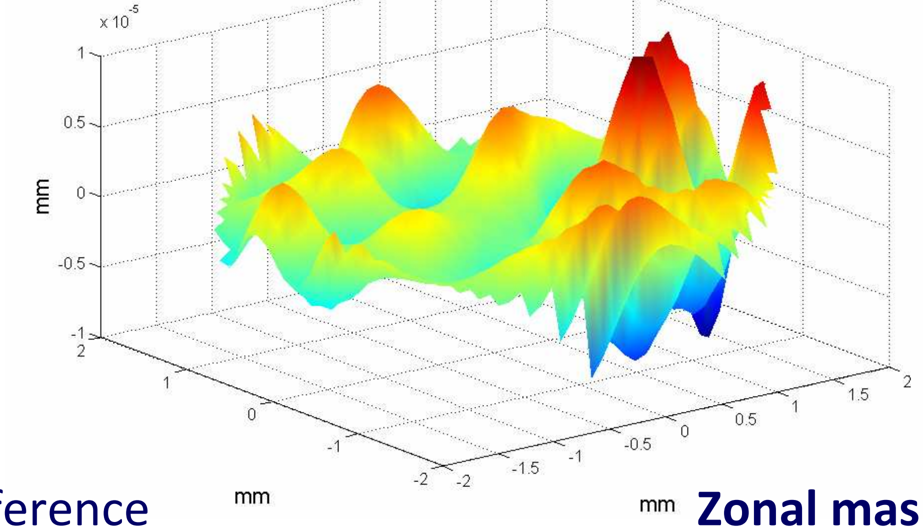
RMSD Sphere+Franke's function

Pupil diameter = 4 mm N=41 ➔ M_{opt}=11

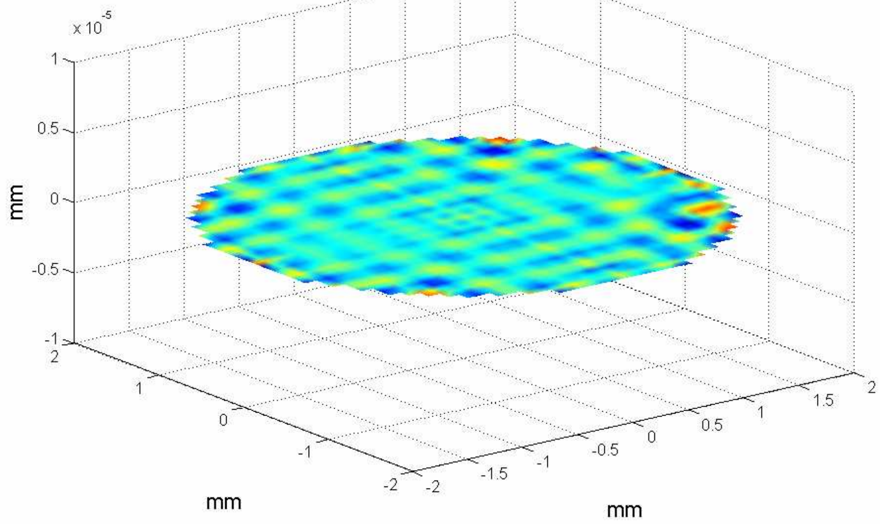


Differences Sphere+Franke's function

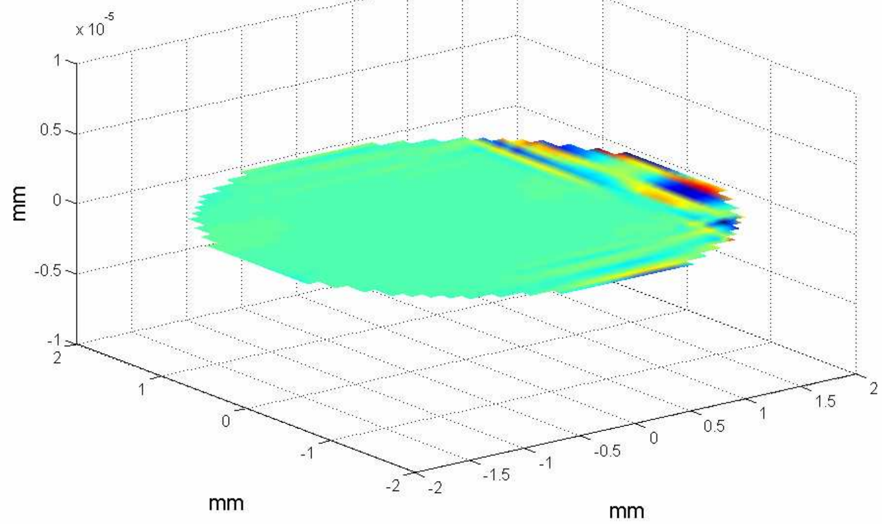
Modal: difference with 120 Zernike polynomials



Zonal mask 21 px: difference with 96 Zernike polynomials



Zonal mask 11 px: difference with 43 Zernike polynomials



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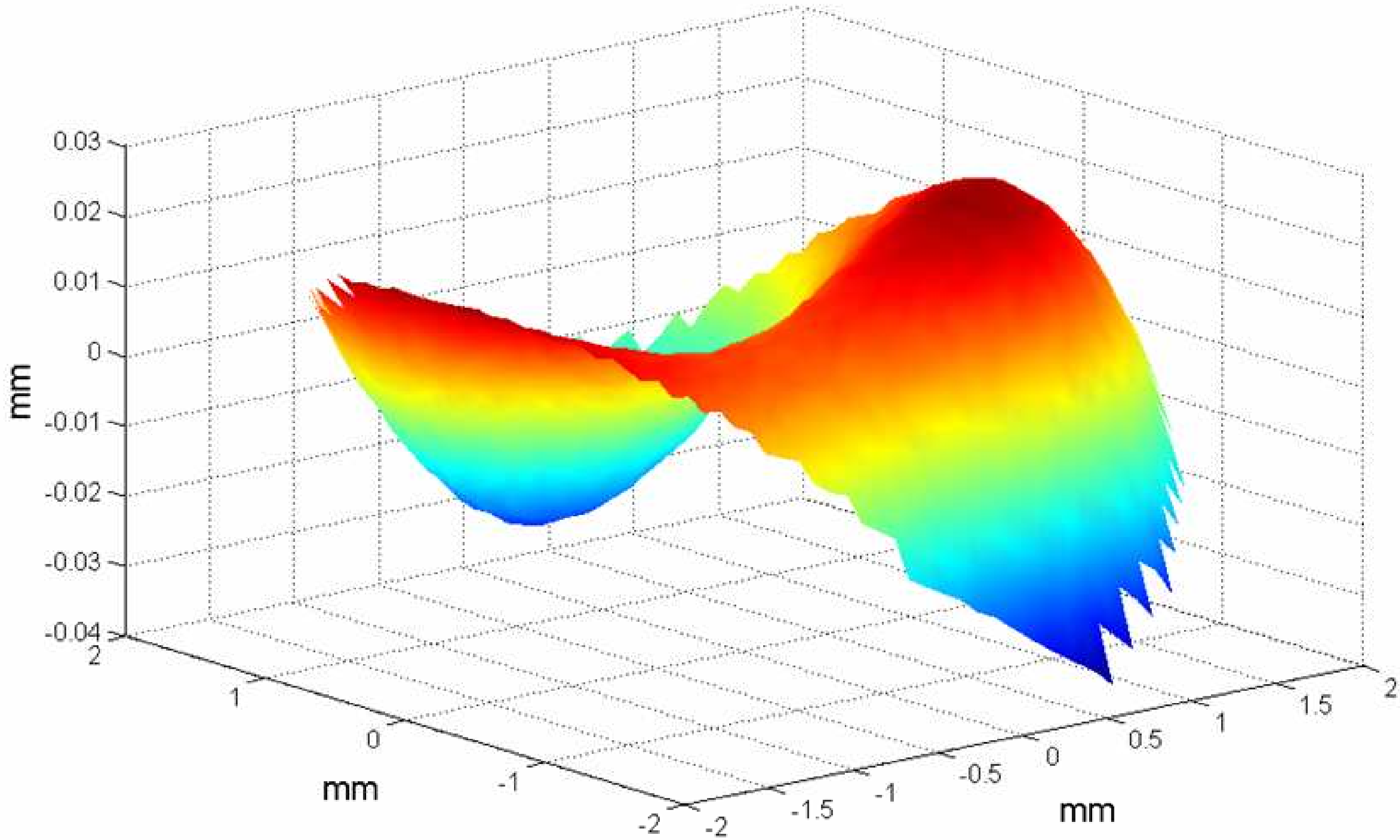
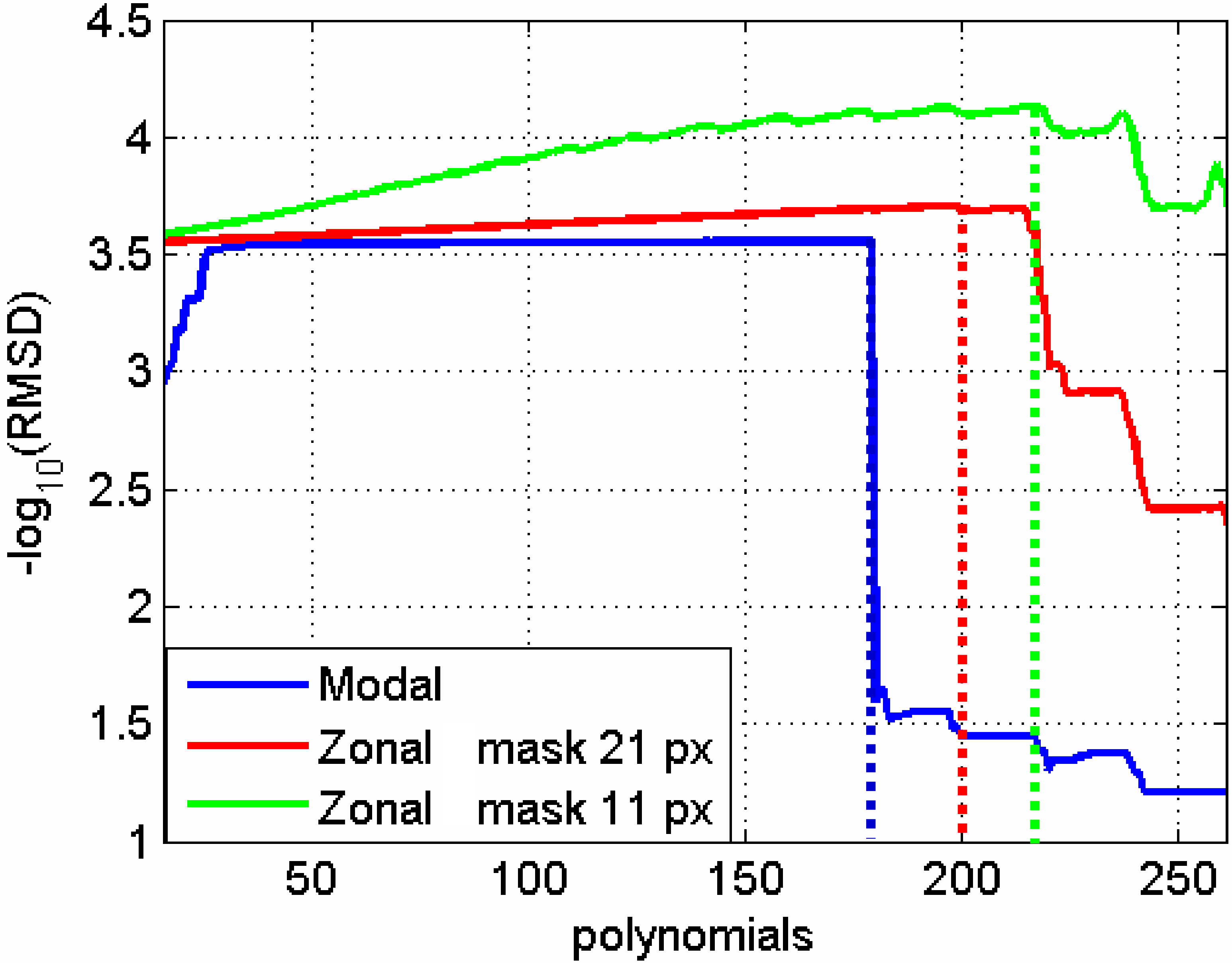
Real irregular surface: Keratoconus

Keratoconus height data - sphere



RMSD keratoconus

Pupil diameter = 4 mm N=41 ➔ M_{opt}=11



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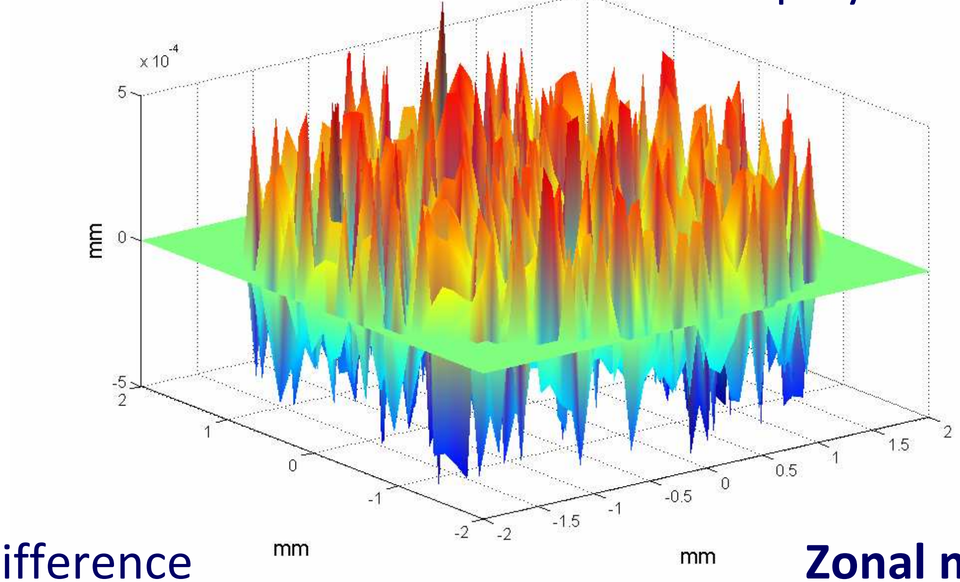
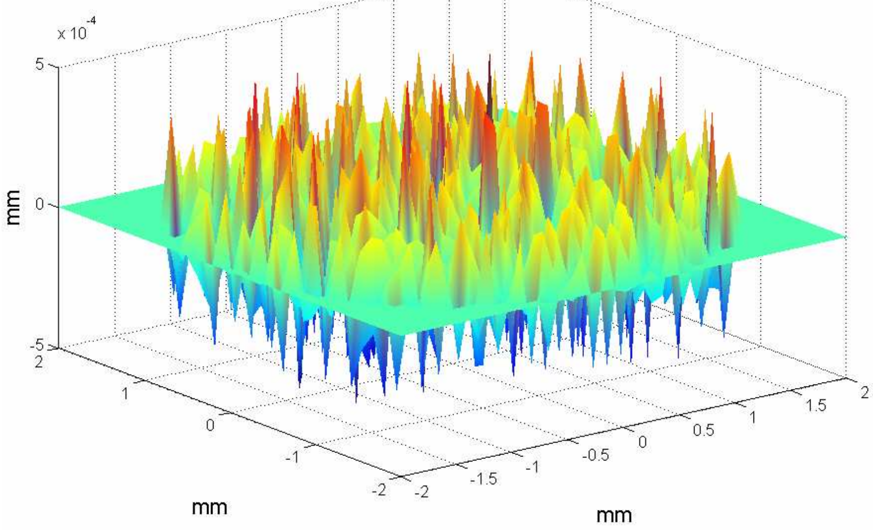
Differences keratoconus

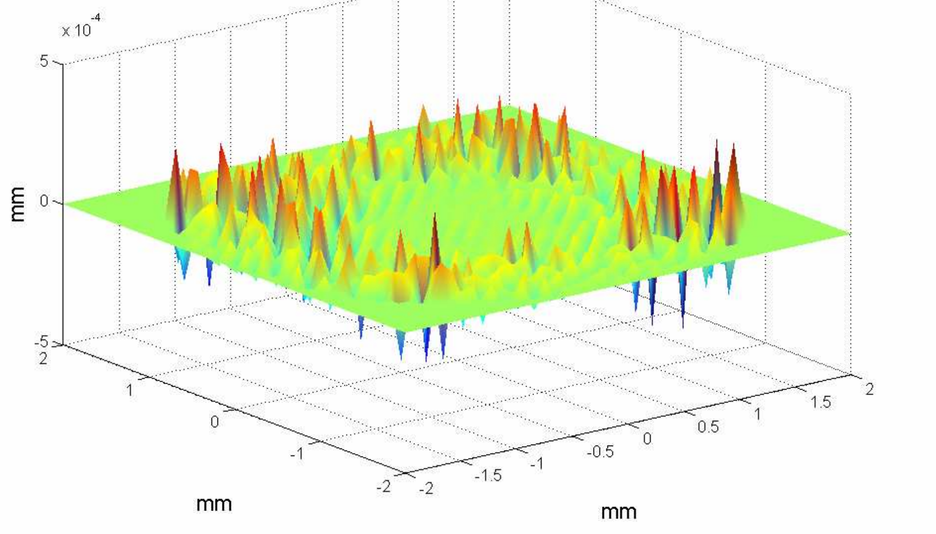
Modal: difference with 178 Zernike polynomials



Zonal mask 21 px: difference with 196 Zernike polynomials



Zonal mask 11 px: difference with 217 Zernike polynomials



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Conclusions

- Better results than modal fit for low order Zernike polynomials.
- The central surface part is better evaluated than the outer parts, since calculation is more intensive in this zone and not affected by peripheral irregularities
- Diminishes the influence of smooth areas over irregular zones and vice versa

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